(4.53) Let $u = u(x,y,z)$, then prove that $\nabla u \cdot d\vec{r} = du$.

Soln:→ we know that

$$\vec{r} = \hat{i}x + y\hat{j} + z\hat{k}$$

$$\therefore d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz \longrightarrow (1)$$

$$\nabla u = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)u$$

$$\nabla u = \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z} \longrightarrow (2)$$

$$L.H.S. = \nabla u \cdot d\vec{r}$$

$$= \left(\hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}\right) \cdot \left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$= \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$L.H.S. = du = R.H.S.$$

(4.59) Let $f = (x-1)^2 + y^2 + (z+2)^2 - 9$

Normal vector $= \nabla f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left[(x-1)^2 + y^2 + (z+2)^2 - 9\right]$

$$= 2(x-1)\hat{i} + 2y\hat{j} + 2(z+2)\hat{k}$$

At point $(3,1,-4)$

Required vector $= 2(3-1)\hat{i} + 2\times 1\hat{j} + 2(-4+2)\hat{k}$

$$= 4\hat{i} + 2\hat{j} - 4\hat{k}$$

Required unit vector $= \dfrac{4\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{4^2 + 2^2 + (-4)^2}}$

$$= \frac{4\hat{i} + 2\hat{j} - 4\hat{k}}{6}$$

$$= \frac{1}{3}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) \quad \text{Ans}$$

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Q:→ Prove that grad $r^m = m r^{m-2} \vec{r}$

Proof:→ We know that $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \rightarrow (1)$ {Position vector}

$$r = (x^2+y^2+z^2)^{\frac{1}{2}} \rightarrow (2)$$

$$r^m = (x^2+y^2+z^2)^{\frac{m}{2}}$$

L.H.S. = grad r^m $= \nabla r^m = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)^{\frac{m}{2}}$

$$= \frac{m}{2}(x^2+y^2+z^2)^{\frac{m}{2}-1}\left[2x\hat{i} + 2y\hat{j} + 2z\hat{k}\right]$$

$$= m(x^2+y^2+z^2)^{\frac{m-2}{2}}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= m\left[(x^2+y^2+z^2)^{\frac{1}{2}}\right]^{m-2}\vec{r}$$

$$= m\, r^{m-2}\vec{r} = \text{R.H.S.}$$

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